May 9th, 2006 **Name (Please Print)** ______________________

B. Math 2nd year **Optimisation** **Final Exam.**

This is a closed book exam. Maximum possible score is 100. There are Seven problems. **Show all your work.** *Partial credit will be given for partial solutions. Correct answers with insufficient or incorrect work will not get any credit.*

1.(15 points) Consider the following Pareto-optimal decision of a given amount of resources between two agents. Given a weight $\alpha \in (0,1)$, solve the problem:

$$\begin{array}{rc} \text{Maximise} & \alpha x_1 y_1 + (1-\alpha) x_2 y_2 \\ \text{Subject to} & x_1 + x_2 \leq 10 \\ & y_1 + y_2 \leq 5 \\ & x_1 \geq 0, y_1 \geq 0, x_2 \geq 0, y_2 \geq 0 \end{array}$$

Please provide adequate justification for any Theorem that you wish to use.

2.(15 points) Let $I, p_1, p_2 > 0$. Consider the following utility maximisation problem:

$$\begin{array}{rc} \text{Maximise} & x_1 + x_2 \\ \text{Subject to} & I - p_1 x_1 - p_2 x_2 \geq 0 \\ & x_1 \geq 0, x_2 \geq 0 \end{array}$$

Please solve the above problem with adequate justification for any Theorem that you wish to use.

3. (15 points) Let $S = [0,2]$ and $\Theta = [0,1]$. Let $f : S \times \Theta \to R$ be defined

$$f(x,\theta) = \begin{cases} 0 & \text{if } \theta = 0 \\ \frac{x}{\theta} & \text{if } \theta > 0, \text{ and } x \in [0,\theta) \\ 2 - (\frac{x}{\theta}) & \text{if } \theta > 0, \text{ and } x \in [\theta, 2\theta] \\ 0 & \text{if} x > 2\theta \end{cases}$$

Let the correspondence $D : \Theta \to P(S)$ be defined by

$$D(\theta) = \begin{cases} [0, 1-2\theta] & \text{if } \theta \in [0, \frac{1}{2}) \\ [0, 2-2\theta] & \text{if } \theta \in [\frac{1}{2}, 1] \end{cases}$$

Do f and D meet all the conditions of the Maximum Theorem ? Justify. Is it the case that $D^*(\theta) \neq \emptyset$? If yes then determine if it is usc or lsc on Θ

4. (a) (5 points) Define the following terms: (i) Convex subset $S \subset \mathbb{R}^n$ (ii) Extreme points of S.
(b) (10 points) Describe the Dual of the LP and the complimentary slackness conditions:

$$\begin{array}{ll} \text{minimise} & 5x_1 + 4x_2 \\ \text{subject to} & 2x_1 - 3x_2 = 8 \\ & -x_1 + 2x_2 \geq 10 \\ & -x_1 + x_2 \leq 3 \\ & x_1 \geq 0, x_2 \in \mathbb{R} \end{array}$$

(c) (10 points) The following is an intermediatory step of the simplex method.

$$\begin{array}{c|cccccc} 1 & 3 & 0 & \frac{-68}{15} & 0 & 0 \\ \hline & & & & & \\ 2 & 2 & 1 & -3 & 0 & 0 \\ 1 & 1 & 0 & \frac{8}{3} & 1 & 0 \\ 3 & -1 & 0 & \frac{29}{15} & 0 & 1 \end{array}$$

Write down the current basic feasible solution, its basis, and the cost at this basic feasible solution. Is this solution optimal ? If no, then indicate choice of j and l for the next pivoting step (DO NOT PERFORM the next pivoting step).

5. (20 points) Solve the following LP Problem using the Simplex Tableaux, with appropriate explanations (NO credit for doing it otherwise).

$$\begin{array}{ll} \text{minimise} & 2x_1 - x_2 \\ \text{subject to} & -x_1 + x_2 \leq 2 \\ & 2x_1 + x_2 \leq 6 \\ & x_1 \geq 0, x_2 \geq 0 \end{array}$$

6. (10 points) Represent the following game in extensive form and then reduce it to matrix form.

Player A has an Ace and a Queen. Player B has a King and a Joker. The rank precedence is Ace > king > Queen but Joker is peculiar as it is described further.

Each player contributes a Rupee to the pot before the game starts. Each selects one of the cards and reveals them simultaneously.

If B selects the king then the highest card owner wins the pot and the game ends. If B selects the joker and A queen then they share the pot equally and the game ends. If B selects the joker and A the Ace then A may either resign(in which case B get the pot) or may demand a replay. If there is a replay then each of them puts another rupee into the pot. Now if B selects the joker and A the ace then B wins the pot.